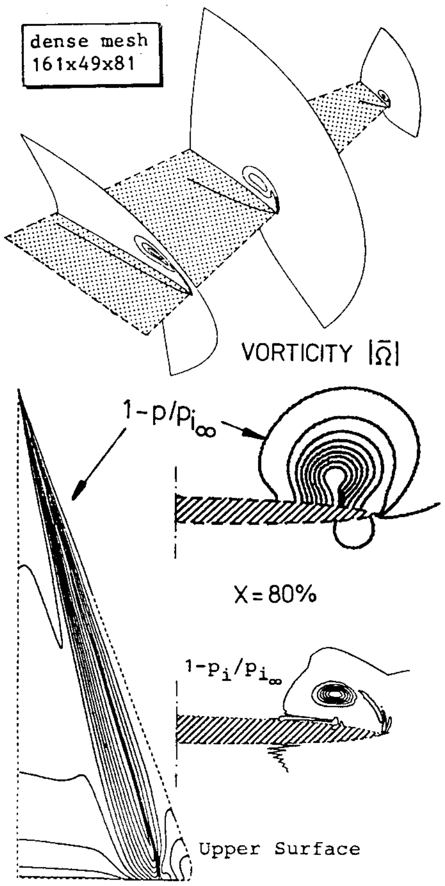

Fig. 2 Vorticity magnitude, static pressure $1-p/p_{i_\infty}$ (increment 0.05), and total pressure $1-p_i/p_{i_\infty}$ (increment 0.05) computed on an O-O mesh of $161 \times 49 \times 81$ points around the Dillner wing ($M_\infty = 0.7$, $\alpha = 15$ deg).

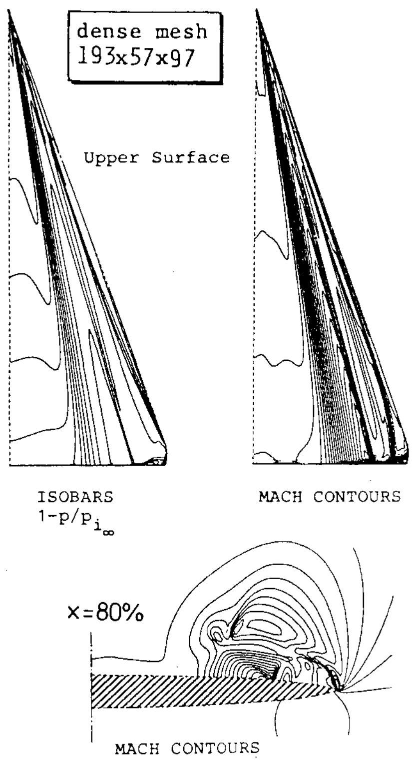

Fig. 3 Iso-Mach contours (increment 0.1) and isobars $1-p/p_{i_\infty}$ (increment 0.05) computed on an O-O mesh of $193 \times 57 \times 97$ points around the Dillner wing ($M_\infty = 1.5$, $\alpha = 15$ deg).

the core of the vortex. This feature of the flowfield is consistent with the experimental findings of Miller and Wood (see Fig. 12 in Ref. 4). Furthermore, we see in the Mach number contours on the upper surface at least two distinct shock waves between the suction peak and leading edge, forming a complex system of shocks and expansion waves being reflected from the apex to the trailing edge. Such phenomena should be expected because the flow, which is supersonic, has to turn abruptly where the vortex sheet leaves the leading edge. And accompanying these phenomena are heavy losses in total pressure.

Conclusions

The dense-mesh solutions presented here have shown that the flowfield around a trapezoidal wing like the M6 can be represented with reasonable accuracy on a standard-size mesh of, say, 50,000 grid points. But if the wing is one of low aspect ratio, like the Dillner delta wing, a much denser grid is required to capture the rich structure of the vortex flow interacting with the shock waves.

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Boundary-Layer Thickness and Base Pressure

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Introduction

RESULTS from base pressure measurements at large boundary-layer thicknesses are rare and the present author had in vain sought such results in order to correlate with his own theory.[1,2] Recently, however, he discovered the measurements of Goecke,[3] which were performed on aft-facing steps in free flight. These experiments included relative boundary-layer thicknesses up to $\delta_2/d = 0.85$, with δ_2 as boundary-layer momentum thickness and d as step height. It was found that the results of Goecke[3] could be well correlated with the theory.[1,2] Since this theory is relatively new and not widely known, its main outline is given first and then it will be used to correlate Goecke's results.

Received Sept. 20, 1984; revision received Nov. 9, 1984.

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Fundamentals of the Theory

Figure 1a illustrates an inviscid supersonic flow past a backward-facing step and Fig. 1b the corresponding flow of a viscous fluid. It is assumed that the freestream Mach number M_∞, the Mach number M_2 at the dead-air boundary, and the base pressure p_B are equal in both cases. The boundary-layer thickness before separation is assumed to be negligible. Then the turbulent boundary layer has no influence on the base pressure. We assume further that, for the flow shown in Fig. 1b, turbulent mixing takes place between the outer flow and the dead-air region, which results in the circulating flow illustrated. The mixing also produces a shear layer that interacts with and modifies the shock as shown in Fig. 1.

The rate of increase of entropy in the flow with viscosity is the sum of two components

$$\Delta S = \Delta S_1 + \Delta S_2 \tag{1}$$

where ΔS_2 is the entropy increase rate due to the shock wave and ΔS_1 the entropy increase rate due to the shear layer. In the frictionless flow (Fig. 1a), the total rate of entropy increase is wholly due to the shock wave.

According to the theory of Oswatitsch[4,5] the following relation is valid:

$$\Delta S = D_B (U_\infty / T_\infty) \tag{2}$$

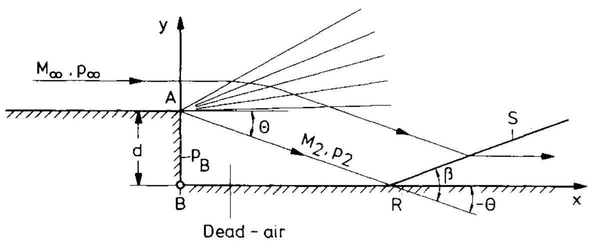

Fig. 1a Inviscid supersonic flow past a backward facing step (A = separation point, R = reattachment point, S = shock wave).

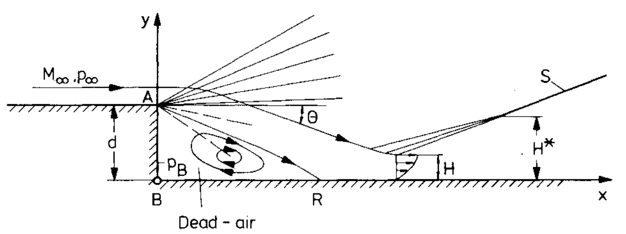

Fig. 1b Supersonic flow of a viscous fluid past a backward facing step (A = separation point, R = reattachment point, S = shock wave, H = shear layer thickness at the shock wave, H^* = effective distance of shock wave from the wall).

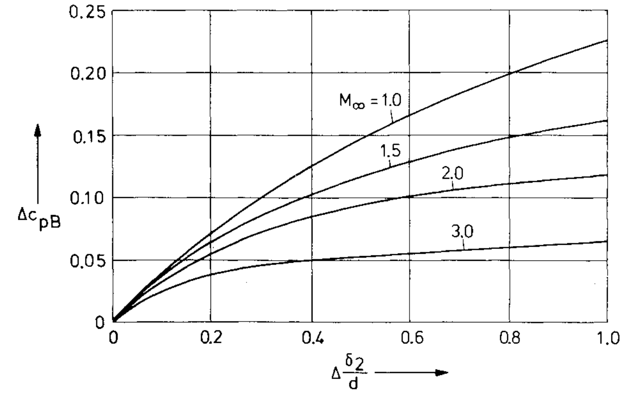

Fig. 2 Theoretical influence of the boundary-layer thickness on the base pressure coefficient for various Mach numbers.

where ΔS is the entropy increase rate, D_B the base drag of the backward-facing step, U_∞ the freestream velocity, and T_∞ the freestream temperature. Since the base pressure—and thus the base drag—is assumed to be equal both for the frictionless and viscous flows, it follows from Eq. (2) that the rate of entropy increase is equal in both cases. Therefore, the flow of entropy in the shear layer of thickness H must be equal to the entropy flow produced by the shock wave of the projected length H^* (see Fig. 1b). For given freestream conditions, this condition is valid for only one value of the base pressure p_B.

By using this criterion, it is possible to derive equations for the calculation of the base pressure of a simple backward-facing step. It is also possible to extend the theory to steps having a boattail, to wedges, and finally also to flows with a thick turbulent boundary layer. For details, Refs. 1 and 2 should be consulted.

In Fig. 2 the theoretical increase of the base pressure coefficient Δc_{pB} with increase of the momentum thickness $\Delta\delta_2/d$ is plotted for various values of the freestream Mach number. Δc_{pB} is

$$\Delta c_{pB} = c_{pB} - (c_{pB})_{0.003} \tag{3}$$

where $(c_{pB})_{0.003}$ is the base pressure coefficient for $\delta_2/d = 0.003$. Likewise,

$$\Delta(\delta_2/d) = \delta_2/d - 0.003 \tag{4}$$

Comparison with Measurements

Figure 3 shows the base pressure coefficient as function of the relative boundary-layer momentum thickness for various Mach numbers. Goecke[1] measured the quantity p_B/p_R, where p_R is a reference pressure. The reference pressure p_R is now assumed to be equal to the freestream pressure p_∞. The base pressure coefficient corresponding to the measured p_B/p_R is then calculated from

$$c_{pB} = (2/\gamma M_\infty^2)(p_B/p_R - 1) \tag{5}$$

The open symbols in Fig. 3 denote experimental values. One can see that the c_{pB} values increase (the pressure drag decreases) with increasing boundary-layer momentum thickness δ_2/d.

The reduction of the measured values to be valid for a boundary-layer thickness of $\delta_2/d = 0.003$ is then performed by using the theory.[1] The results are shown as solid symbols for the three different Mach numbers. For the constant

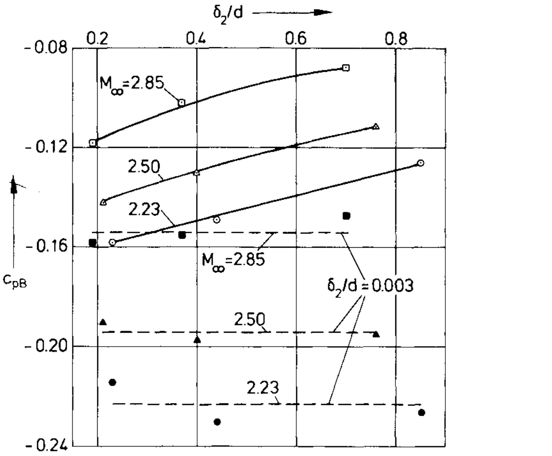

Fig. 3 Base pressure coefficient c_{pB} as function of the relative momentum thickness δ_2/d (open symbols: measurements of Ref. 3, solid symbols: measured values reduced to $\delta_2/d = 0.003$ by using theory of Ref. 1).

boundary-layer thickness $\delta_2/d = 0.003$, the c_{pB} values should also be constant for each Mach number. One can see that this nearly is the case. The deviations from a mean value in each group are relatively small. At $M_\infty \approx 2.85$, the deviations depend partly on the fact that the Mach number is not constant, but varies from $M_\infty = 2.81$ at $\delta_2/d = 0.19$ to $M_\infty = 2.89$ at $\delta_2/d = 0.70$.

There are, of course, several other theories for prediction of the base pressure in two-dimensional supersonic flow. Perhaps the most important are based on the flow model of Chapman[6] and Korst[7] where the basic physical idea is that the base pressure can be predicted if the pressure at the reattachment point (see Fig. 1) is known. Of these theories, we additionally mention only those of Refs. 8-10, which allow the prediction of the influence of the boundary-layer thickness on the base pressure. The theories have been discussed and compared in Ref. 11. It seems that the theory of McDonald[10] agrees at least qualitatively with experiment, whereas Refs. 8 and 9 overestimate the effect of the boundary-layer thickness. Experimental results exist obviously only for $\delta_2/d \leq 0.10$.

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Block-Implicit Calculations of Three-Dimensional Laminar Flow in Strongly Curved Ducts

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Nomenclature

A_p, A_i = coefficients in the finite difference equation
d = width (and height) of the duct
p = pressure
r, θ, z = coordinate distances
S = integrated source term
v_b = bulk axial velocity
v_r, v_θ, v_z = linear velocities in r, θ, and z directions, respectively
x = distance in the straight sections
ρ = fluid density
μ = fluid viscosity

Received April 11, 1984; revision received Jan. 16, 1985.

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Introduction

THE understanding of flow development in ducts with longitudinal curvature is of interest in the design of aircraft intakes, turbomachinery passages, and heat exchange equipment. A number of previous studies[1-5] have attempted to solve numerically the partial differential equations appropriate to the curved duct configuration. Experimental studies quantifying the velocity and pressure fields also have been made.[4,6,7] A review of flow development in curved circular pipes was recently made by Berger et al.[8]

This Note describes the numerical solution of the elliptic partial differential equations governing three-dimensional fluid flow in strongly curved rectangular ducts by the use of a fully coupled block-implicit solution algorithm. Calculations have been made for the configuration of Humphrey et al.[4] It is observed that the coupled solution of the momentum and continuity equations converges rapidly and reduces the required computing time by a factor of 2.5 over a decoupled solution such as that by Humphrey et al.[4] The flowfield calculated here is in good agreement with the experimental data and with previous calculations.[4] The flow separation on the outer sidewall is well predicted, in conformity with earlier observations.

Equations Solved

Theoretical studies of curved duct flow development need the solution of complete elliptic Navier-Stokes equations governing steady three-dimensional flows. At small curvatures, however, the flow may be considered parabolic[9] or partially parabolic[10] and streamwise diffusion may be neglected. The present study is concerned with the calculation of flow in strongly curved ducts with regions of axial flow reversal. Consequently, the following three-dimensional elliptic equations are solved.

$$\rho\left(v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right) \tag{1}$$

$$\rho\left(v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right) \tag{2}$$

$$\rho\left(v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\nabla^2 v_z \tag{3}$$

$$\frac{\partial v_z}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0 \tag{4}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

The above equations are solved with boundary conditions appropriate to the experimental configuration.